

Symmetry of Japanese Kanji Lexical Productivity on the Left- and Right-hand Sides

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Abstract: Japanese kanji combine with other kanji to produce various two-kanji compound words. First, the present study examined whether the extent of left-hand and right-hand productivity of the Japanese 1,945 basic kanji abides by an ‘honest’ distribution. The result showed that kanji compound building (or kanji lexical productivity) was depicted by a birth-and-death process leading to the negative binomial and/or the Waring distribution. Second, the study investigated whether these basic kanji display symmetry on the left- and right-side lexical productivity. Analysis of these kanji suggested that although each kanji displayed symmetry in lexical productivity, there is no tendency among the basic kanji to produce their compound words to the same extent on the left or the right side on the whole.

Key words: Japanese kanji, kanji lexical productivity, birth-and-death process, symmetry

1. Two-kanji compound words in a Japanese Dictionary

In the written Japanese, *kanji*, which adopted Chinese characters for expressing various words, often combine with another kanji to produce a new meaning. In this sense, the unit of kanji refers to ‘morpheme’, the smallest unit of meanings. Yokosawa and Umeda (1988) reported that approximately 70 percent of 51,962 words listed in a particular Japanese dictionary were composed of two kanji or two morpheme combinations. In 1981, the Ministry of Education, Science, Sports and Culture, Government of Japan (hereafter simply called the ‘Japanese Ministry of Education’ except in quotes) published a list of the *1,945 Basic Japanese Kanji* (for detailed information, see Kato, 1989; Yasunaga, 1981). This list established a standard for kanji usage in printed and written Japanese texts (Ministry of Education, Science, Sports and Culture, Government of Japan, 1987, 1998). The National Institute for Japanese Language (1976) conducted a survey on the frequency of kanji in print, and found that 2,000 kanji encompassed 99.6 percent of all kanji used in three major Japanese newspapers, *Asahi*, *Mainichi* and *Yomiuri* published during the year 1966. Although the 1,945 Basic Japanese Kanji and the 2,000 kanji mentioned above were not identical, it is roughly estimated that these basic kanji cover approximately 99 percent of all kanji used in Japanese newspapers. Combining this figure with 70 percent of two-kanji compound words in a dictionary, 1,945 kanji will provide us a reasonable estimate on Japanese kanji and their productivity.

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2. Counting of kanji lexical productivity and purpose of the present study

As Yokosawa and Umeda (1988) pointed out, the majority of lexical items in a Japanese dictionary are words consisting of two kanji units. The number of lexical productivity is simple counting (i.e., type frequency, not token frequency) of all possible two kanji combinations. Interestingly, various complex compound words are often constructed by two kanji compound words such as 経済政策 (/keizai seisaku/², ‘economic policy’), 宇宙遊泳 (/utjuR juRei/, ‘space walk’), and 冷凍食品 (/reitoR sjokuhiN/, ‘frozen food’). Furthermore, the word ‘frozen food’ can be combined with another two-kanji compound word 貯蔵 (/tjo zoR/, ‘storage’) to make a six-kanji compound word, 冷凍食品貯蔵 (/reitoR sjokuhiN tjozoR/, ‘storage of frozen food’) consisting of three two-kanji compound words. Thus, two-kanji compound words are a sensible unit to count how many words each kanji can produce.

A single kanji can produce two-kanji compound words in two ways, produced by the combination of kanji placed on the left-hand and right-hand side positions of two-kanji compound words. For example, the kanji 学 /gaku/ meaning ‘to learn’ or ‘learning’ is combined with another kanji on the right-hand side position such as in 学校 (/gaQ koR/, ‘school’), 学生 (/gaku sei/, ‘student’) and 学者 (/gaku sja/, ‘scholar’). Combinations with other kanji on the left-hand side position are also possible such as in 入学 (/njuR gaku/, ‘school admission’), 文学 (/bun gaku/, ‘literature’) and 私学 (/si gaku/, ‘private school’). Kanji productivity of two-kanji compound words refers to the unit of one kanji combined with another kanji to create two-kanji compound words. Therefore, the concept of this term could be understood as a linguistic concept of kanji ‘lexical productivity’ (details see, Hayashi, 1987; Nomoto, 1989; Nomura, 1988, 1989), which can be calculated by the left-side, right-side and both sides together. The present paper investigated two questions: (1) how two-kanji compound words were produced by a single kanji on the left-hand and the right-hand sides, and (2) how symmetric they are on both sides.

3. Kanji lexical productivity

The present study used a lexical corpus of 341,771 words that was established from newspapers containing 287,792,797 words by Amano and Kondo (2000), all of which were taken from the *Asahi Newspaper* printed from 1985 to 1998. At present, this is the largest and the most up-to-date word corpus created from calculating the word frequency of occurrence in Japanese written texts. For counting kanji lexical productivity, the programming language of MacJPerl 5.15r4J for Macintosh was used to run a calculation procedure. Type frequency counts of kanji lexical productivity on the left-hand, right-hand and both sides were arranged for all the 1,945 basic kanji. The beginning 10 kanji of the raw data are presented in Table 1.

² The pronunciation in this paper is transcribed using Japanese phonemic symbols which indicate three special sounds in Japanese: /N/ for nasal, /Q/ for geminate and /R/ for long vowel.

Table 1

The first ten kanji and their lexical productivity on the left- and right-hand sides

No.	Kanji	In phonemes	Left-hand side	Right-hand side	Both sides together
1	亜	/a/	3	13	16
2	哀	/ai/	1	19	20
3	愛	/ai/	28	55	83
4	悪	/aku/	35	80	115
5	握	/aku/	4	4	8
6	圧	/atu/	42	21	63
7	扱	/atu/	1	1	2
8	安	/aN/	24	40	64
9	案	/aN/	49	6	55
10	暗	/aN/	5	40	45

Note: A common kanji pronunciations are used to arrange kanji in this table.

Each of the chosen kanji develops its own individual left and right lexical productivity. For example, the eighth kanji 安 (/aN/, 'rest', 'relax', 'cheap', 'peaceful', etc.) produced 24 two-kanji compound words by adding kanji on the left-hand side while producing 40 compounds by adding kanji on the right-hand side. The total lexical productivity becomes 64. By counting the number of kanji with x compounds, the results presented in Table 2 for kanji on the left-hand side and in Table 3 for kanji on the right-hand side. Two-step investigation is conducted using data of kanji lexical productivity in the present study. The first step is to conjecture the mechanism generating the distribution of compounds. The second step is the eventual modification of the result. This second step must be made because the basic kanji were taken from a ready list, not at random from the dictionary. Thus the bias can be quite systematic.

Table 2

Distribution of productivity of the 1,945 basic kanji on the left-hand side

x	fx	X	fx	x	fx	x	fx
0	108	35	10	71	2	112	1
1	135	37	14	72	1	114	1
2	109	38	16	73	1	116	1
3	128	39	5	75	5	117	1
4	82	40	9	76	1	118	4
5	84	41	13	77	3	120	2
6	84	42	6	78	2	121	2
7	61	43	8	79	2	122	1
8	73	44	10	80	2	123	1

9	67	45	9	82	4	126	1
10	59	46	8	83	2	127	1
11	53	47	5	84	2	128	1
12	48	48	8	85	2	129	1
13	49	49	6	86	1	130	1
14	36	50	5	87	1	131	2
15	37	51	2	88	1	132	1
16	37	52	7	89	2	135	2
17	39	53	9	91	1	136	1
18	33	54	3	92	1	138	1
19	29	55	6	93	2	140	1
20	17	56	2	94	1	142	1
21	34	57	5	96	2	146	1
22	28	58	3	97	1	148	1
23	15	59	5	98	1	151	1
24	17	60	3	99	1	153	1
25	22	61	3	100	1	162	1
26	21	62	3	101	1	170	1
27	19	63	3	102	1	171	1
28	16	64	6	103	1	180	1
29	13	65	6	104	1	213	1
30	16	66	3	106	4	246	1
31	15	67	8	108	1	268	1
32	13	68	3	109	2		
33	8	69	3	110	1		
34	11	70	3	111	1		

Note: x = number of compounds, f_x = number of kanji producing x compounds.

Table 3
Distribution of productivity of the 1,945 basic kanji on the right-hand side

x	f_x	X	f_x	x	f_x	x	f_x
0	72	33	17	67	3	111	2
1	91	34	16	68	1	114	1
2	115	35	7	69	2	116	1
3	121	36	13	70	4	118	1
4	90	37	13	71	4	119	1
5	93	38	13	72	5	120	1
6	92	39	7	73	3	126	1
7	87	40	11	74	1	127	1
8	80	41	11	75	3	132	1
9	61	42	15	76	2	135	2
10	78	43	9	77	6	137	2
11	51	44	12	78	1	138	3
12	57	45	4	79	3	145	1
13	49	46	6	80	4	146	1
14	51	47	10	82	1	148	1

15	34	48	5	83	2	149	1
16	45	49	7	86	1	151	1
17	32	50	8	87	3	154	2
18	35	51	5	88	1	156	1
19	31	52	6	90	2	157	1
20	29	53	4	91	1	158	1
21	22	54	8	92	1	160	1
22	21	55	4	95	2	163	1
23	18	56	2	96	1	170	1
24	22	57	4	97	4	179	1
25	11	58	1	99	1	194	1
26	21	59	6	101	1	195	1
27	15	60	3	103	2	293	1
28	17	61	5	104	1	350	1
29	12	62	4	106	3	399	1
30	14	63	5	107	1		
31	15	64	3	109	1		
32	15	66	1	110	1		

Note: x = number of compounds, fx = number of kanji producing x compounds.

4. A birth-and-death process of kanji compound words

Lexical productivity in languages, consisting of two kanji in the present study, always faces to the birth of new forms (e.g., compounds) and their death. The process of the birth and death has two general features in languages. First, this process is incessant in any language since newly-formed words are influenced steadily by various linguistic environments. As a result, some newly-produced words die immediately, some live longer, and some survive for a very long time. Second, this process is, nevertheless, in steady state and is balanced by language self-regulation (Köhler 1986). Thus, having these two features together, lexical productivity is consistently modeled in the manner of an incessant steady-state birth-and-death process (Altmann 1985; Wimmer, Altmann 1995).

Considering f_x the number of kanji building exactly x compounds, P_x is the relative number corresponding to f_x , or the probability in the model. A kanji leaves class x if either a compound “dies” (the kanji goes in class $x-1$) or a new compound arises (it goes to class $x+1$). A kanji enters class x if in class $x-1$ a new compound is born or in the class $x+1$ a compound dies. Let the birth ratio be λ_x and the death ratio μ_x . Then, considering the probability of birth or death of two or more compounds at a time as zero, the process can be written in form of simple equations:

$$(1) \quad \begin{aligned} \lambda_0 P_0 &= \mu_1 P_1 \\ (\lambda_x + \mu_x) P_x &= \lambda_{x-1} P_{x-1} + \mu_{x+1} P_{x+1}, \quad x = 1, 2, \dots \end{aligned}$$

The equation (1) can be solved stepwise but the solution is known as

$$(2) \quad P_x = P_0 \frac{\lambda_0 \lambda_1 \dots \lambda_{x-1}}{\mu_1 \mu_2 \dots \mu_x} .$$

Using simple linear functions for the rates, namely

$$\begin{aligned}\lambda_x &= a+bx \quad (a = \text{constant birth coefficient, } b = \text{coefficient of assertion against } x \\ &\quad \text{rivals)} \\ \mu_x &= cx \quad (c = \text{death coefficient}).\end{aligned}$$

The equation (3) is obtained by substituting them in (2)

$$(3) \quad P_x = P_0 \frac{a(a+b)(a+2b)\dots[a+(x-1)b]}{x!c^x}.$$

The rest is simple manipulation. Factoring out b and requiring $c > b$ (because of convergence), and substituting

$$\begin{aligned}b/c &= q \quad (0 < q < 1) \\ a/b &= r\end{aligned}$$

the equation (4) is obtained:

$$(4) \quad P_x = P_0 \frac{r(r+1)\dots(r+x-1)}{x!} q^x = P_0 \binom{r+x-1}{x} q^x.$$

Since P_0 is the normalizing constant following from $\sum P_x = 1$, we obtain at last (with $p = 1-q$)

$$(5) \quad P_x = \binom{r+x-1}{x} p^r q^x, \quad x = 0, 1, 2, \dots$$

representing the usual negative binomial distribution, symbolized as $NB(r,p)$. Now fitting this distribution to the data, it can be seen easily that it is not adequate because of the possible bias. The data can even be smoothed by pooling the empirical classes but we rather seek a modified model. The model can be found in different ways. The study will present two of them.

4.1. Fitting the mixed negative binomial distribution for kanji lexical productivity on the left-hand side

Since the kanji are taken from the ready list which may contain different classes of kanji – whatever kinds of classes may be concerned – it is, in the first step possible to add to the equation (5) a second component with different parameters and mixing the components in different proportions, say α and $1-\alpha$. Considering this specification, we obtain from the equation (5) the model in the form

$$(6) \quad P_x = \alpha \binom{r+x-2}{x-1} p_1^r q_1^{x-1} + (1-\alpha) \binom{r+x-2}{x-1} p_2^r q_2^{x-1}, \quad x = 1, 2, 3, \dots$$

where p_1, p_2, r and α are coefficients interpreted above. In our case $k = 1.2269, p_1 = 0.1159, p_2 = 0.0270, \alpha = 0.672$. As can be seen, the Chi-square is 121.52 which with 143 DF signalizes a very good fit, $P = 0.90$. The last theoretical value (at $x = 268$) was computed as $1 - \sum_{x=0}^{267} P_x$. The graphical representation displaying it can be found in Fig. 1. The observed and theoretical values are shown together with class pooling $NP_x > 1$ in Table A of the Appendix.

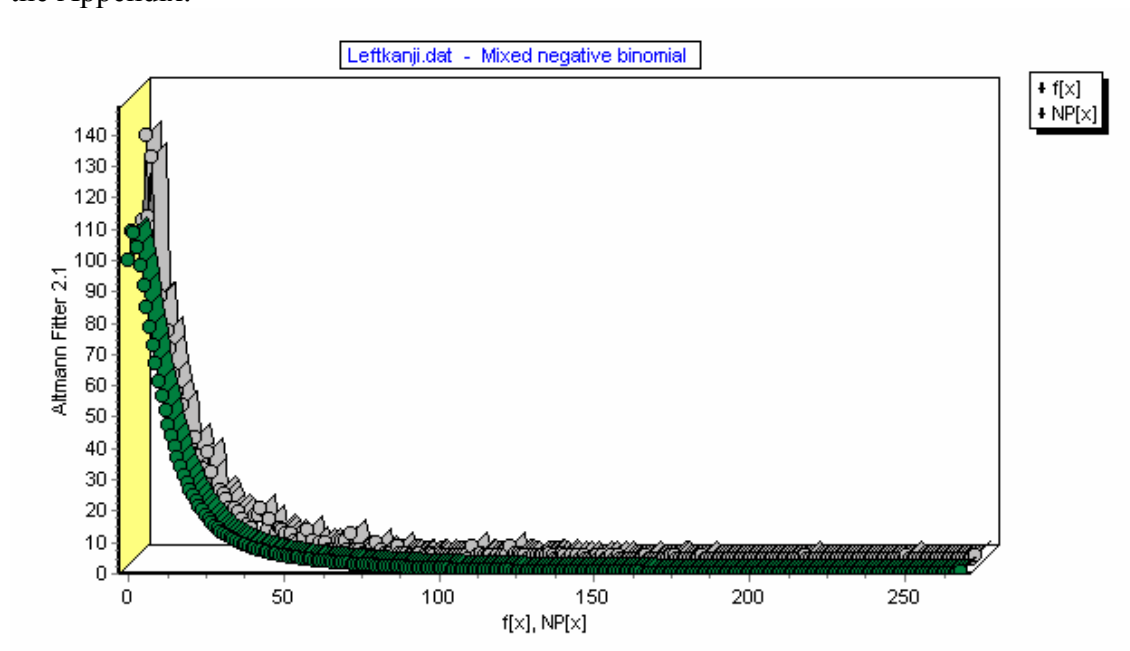


Fig 1. Fitting the mixed negative binomial distribution to left-hand kanji compounding

4.2. Fitting the Waring distribution for kanji lexical productivity on the left-hand side

Another method is the randomization of the parameter p yielding a compound distribution. One of the many possibilities is, symbolically

$$(7) \quad NB(r,p) \underset{p}{\wedge} beta(b,1)$$

where $beta(b,1)$ is the beta distribution with parameters b and 1, that is

$$(8) \quad f(p) = \frac{p^{b-1}}{B(b,1)}, \quad 0 < p < 1, \quad q = 1 - p.$$

Multiplying (5) by (8) and integrating the product according to $p \in \langle 0,1 \rangle$, one obtains the Waring distribution, introduced in linguistics by G. Herdan, yielding

$$(9) \quad P_x = \frac{b}{b+r} \frac{r^{(x)}}{(b+r+1)^{(x)}}, \quad x = 0, 1, 2, \dots$$

However, this distribution can be obtained directly from the birth-and-death process with

birth-rate $\lambda_x = r+x$ and death-rate $\mu_x = b+r+x$. This distribution holds equally well for the data in Table 2: Parameters: $b = 3.4961$, $r = 53.6656$, $DF = 136$, Chi-square = 124.70, $DF = 136$, $P = 0.75$. Thus, one can derive the Waring distribution either directly from the birth-and-death process or indirectly by randomisation. Assuming that more extended examination could conceal some surprise, we preliminarily adhere to the negative binomial distribution.

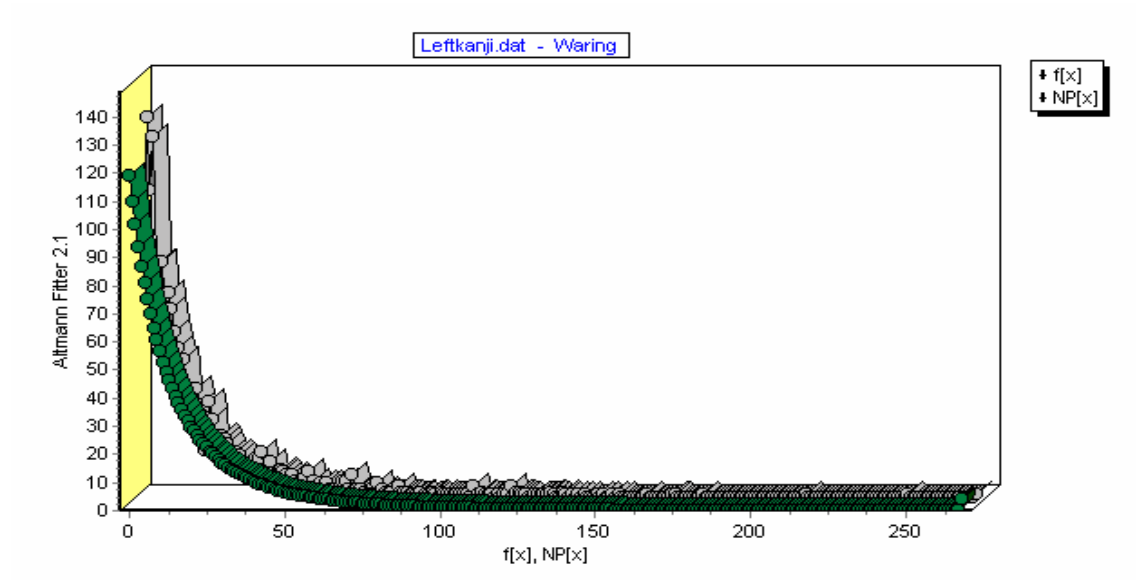


Fig. 2. Fitting the Waring distribution to left-hand kanji compounding

4.3. Kanji lexical productivity on the right-hand side

For the right-hand compounding in Table 3, the mixed negative binomial yields a $X^2 = 134.56$, $DF = 136$, $P = 0.52$, the fitting is shown graphically in Fig. 3. The Waring distribution (with parameters $b = 3.4224$, $r = 50.9697$) is somewhat weaker ($X^2 = 162.19$, $DF = 137$, $P = 0.07$) but still acceptable. Different pooling of data can improve the fitting (up to $P = 0.11$). In Figure 4, one finds the fitting of (6) displayed graphically.

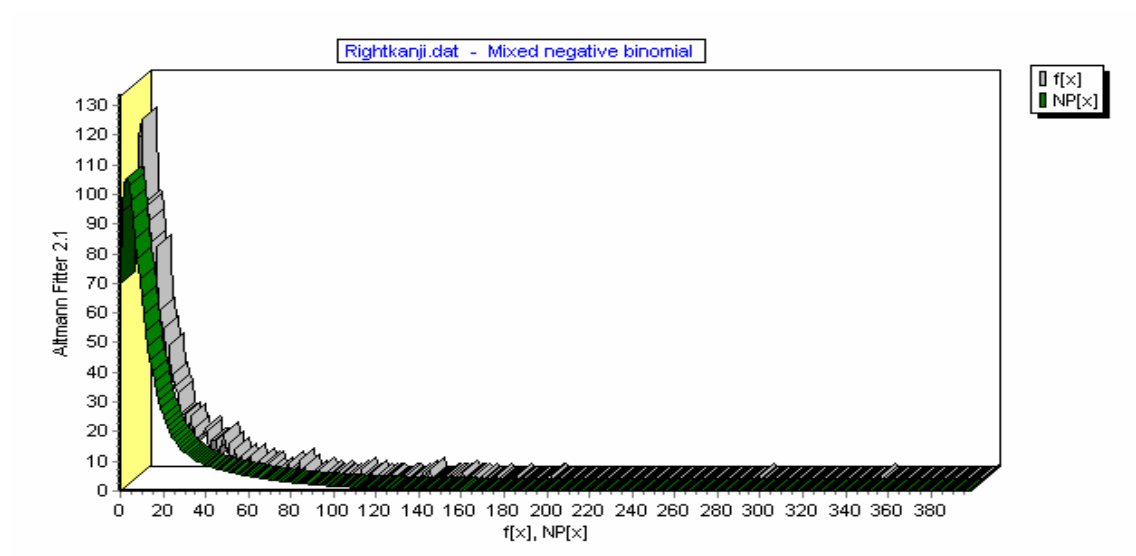


Fig. 3. Fitting the mixed negative binomial d. to right-hand kanji compounding

It must be remarked that the pooling of theoretical values has been performed so that each class contained at least $NP_x > 1.00$. The empirical values were pooled a posteriori but the computation was made for each frequency class explicitly.

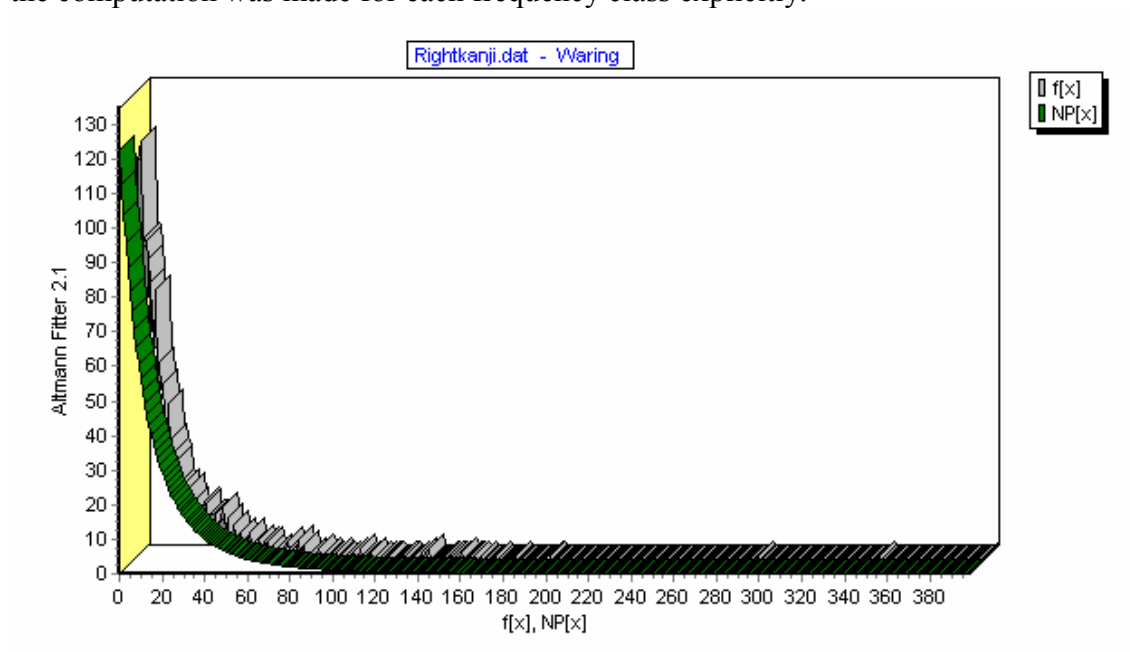


Fig. 4. Fitting the Waring distribution to right-hand kanji compounding

5. Symmetry of kanji lexical productivity

Looking at Table 1, one would intuitively say that left-hand and right-hand side compounding is not symmetrical. In order to test the symmetry (a), one can use for individual kanji the binomial test setting $p = 1/2$, $n = \text{total compounding}$, and compute the probability of, say, the left-hand or the more extreme x . (b) For the whole table one can use the Bowker-test for symmetry (Bowker 1948), leaving out all kanji for which $x_{\text{left}} = x_{\text{right}}$ or simply perform the test for homogeneity of two sides. (c) Making a quadratic contingency table of the data in Table 1, one can test the marginal homogeneity using Stuart's test (Stuart 1955).

But if one constructs a two-dimensional distribution with $X = \text{left-hand}$, $Y = \text{right-hand compounding}$, one obtains an enormous matrix which is neither well processable nor very interesting because the great majority of cells is empty. Thus, pooling is necessary. However, in that case, some differences between left and right compounding (within the pooling interval) disappear and the problem of symmetry gets a new face. Problem (a) is eliminated, problem (b) changes thoroughly, problem (c) will display greater homogeneity, but a new aspect arises, namely (d) the possibility to test whether the diagonal of the contingency matrix is preferred; that is, whether there is a tendency to make left-hand and right-hand compounding similar or not. Using test (a), one gets a result concerning individual kanjis, test (b) ignores the diagonal, but test (d) can show whether there is a tendency to make the productivity on the two sides of kanji similar (not identical).

5.1. The compounding symmetry of individual kanji

In Table 1, kanji No. 1 has 16 compound words out of which it is 3 times on the left side (i.e., it is *progressive*) and 13 times on the right side (i.e., it is *regressive*) of the com-

pounds. One can ask whether this ratio is random or whether there is a tendency to build compounds of a special kind. If there is no tendency, the probability that the compounding part will stay right or left is $p = 0.5$. One finds n (here 16) compounds. What is the probability that exactly x compounds will be progressive (regressive respectively)? Since there are merely two possibilities, the compounds are distributed binomially; thus,

$$(10) \quad P_x = \binom{n}{x} p^x q^{n-x} = \binom{n}{x} (1/2)^n$$

in this case. However, we need the sum of all probabilities being smaller than 0.025 or greater than 0.975; that is, we seek a number k_1 for which $P(X \leq k_1) \leq 0.025$ and a number k_2 for which $P(X \geq k_2) \leq 0.025$. These can be computed by means of the criteria

$$(11) \quad P(X \leq k_1) = \sum_{x=0}^{k_1} \binom{n}{x} (1/2)^n \leq 0.025$$

$$(12) \quad P(X \geq k_2) = \sum_{x=k_2}^n \binom{n}{x} (1/2)^n \leq 0.025.$$

Thus, we obtain a kind of confidence interval $[k_1; k_2]$ within which the basic kanji is symmetric, and below or above the interval there is a significant tendency to construct progressive or regressive compounds.

For our example, we have $n = 16$. Computing the probabilities from 0 according to (11) we see that $k_1 = 4$ and according to (3) $k_2 = 12$ because the sum of the probabilities from $x = 0$ to $x = 3$ is smaller than 0.025, and that from 13 to 16 is smaller than 0.025, i.e. our interval is $[4; 12]$. Since kanji No.1 has 13 regressive compounds, it lies outside of the interval and can be classified as a compositionally regressive kanji.

Consider kanji No. 6 with $n = 63$ compounds. The interval is $[24; 39]$; since the numbers in Table 1 lie outside this interval and the left-hand side productivity is greater, this kanji has a progressive compounding. However kanji No. 8 with $n = 64$ and interval $[24; 40]$ lies still in the interval and can be classified as symmetric. The other values are shown in the last column of Table 4. All intervals for $n = 6$ to 400 are in the Appendix, Table C.

An asymptotic test for symmetry of individual kanji can be performed as follows: Let the number of left-hand side compounds be n_L , that of right-hand side ones n_R and $n_L + n_R = n$. Then, under the hypothesis of equality of both sides, the expected value is $n/2$. The asymptotic Chi-square criterion is

$$(13) \quad X^2 = \frac{(n_L - \frac{n}{2})^2}{\frac{n}{2}} + \frac{(n_R - \frac{n}{2})^2}{\frac{n}{2}} = \frac{(n_L - \frac{n}{2})^2}{\frac{n}{2}} + \frac{(n - n_L - \frac{n}{2})^2}{\frac{n}{2}}$$

$$= \frac{(n_L - n_R)^2}{n_L + n_R}$$

which is distributed as a chi-square with 1 degree of freedom. At the $\alpha = 0.05$ level it must be greater than 3.84 in order to be significant. For example, let $n_L = 2$ and $n = 10$, then (13) yields

$$X^2 = \frac{(2-8)^2}{2+8} = 3.6$$

which is not significant, but for $n_L = 1$, we obtain $(1-9)^2/10 = 6.4$ which is significant. In most cases this test yields the same results as the exact (binomial) test. Instead of n_L one can insert n_R . The interpretation is the same.

All the 1,945 kanji were tested for symmetry. There were 227 kanji (11.67% of the total) with less than 5 of the total kanji lexical productivity putting both the left-hand and right-hand sides together. Excluding these kanji for symmetry, 902 kanji (46.38%) were judged to be symmetric represented by 'S' in Table 4 for the examples of ten kanji. When the left-hand side productivity was greater than the right-hand side, a kanji was judged as progressively asymmetric presented by 'P'. 403 kanji (20.72%) fell into this category. When the right-hand side productivity was greater than the left-hand side, a kanji was judged as regressively asymmetric presented by 'R'. 413 kanji (21.23%) were counted in this category. Simply looking at the percentages, a set of the whole 1,945 basic kanji seems to display symmetric pattern of lexical productivity, being 20.72% for progressive, 46.38% for symmetry and 21.23% for regressive. The next section investigates the pattern of the 1,945 basic kanji.

Table 4
Symmetric classification of the first ten kanji in lexical productivity

No.	Kanji	In phonemes	Left-hand side	Right-hand side	Both sides together	Classification
1	亜	/a/	3	13	16	R
2	哀	/ai/	1	19	20	R
3	愛	/ai/	28	55	83	R
4	悪	/aku/	35	80	115	R
5	握	/aku/	4	4	8	S
6	圧	/atu/	42	21	63	P
7	扱	/atu/	1	1	2	S
8	安	/aN/	24	40	64	S
9	案	/aN/	49	6	55	P
10	暗	/aN/	5	40	45	R

5.2. The symmetry of the entire compounding of 1,945 basic kanji

In order to ascertain whether the whole field of basic kanji displays symmetric compounding, one simply builds the sum of (13) which is identical with Bowker's test for symmetry in a contingency table, i.e.

$$(14) \quad X^2 = \sum_{i=1}^K \frac{(n_{L,i} - n_{R,i})^2}{n_{L,i} + n_{R,i}}$$

where K is the number of comparisons (here 1945). The number of degrees of freedom is $DF = (K - \text{number of cases where } n_L \text{ and } n_R \text{ are equal})$.

An asymptotically equivalent test can be derived using information statistics, namely

$$(15) \quad 2I = 2 \sum_{i=1}^K \sum_{j=1}^2 n_{ij} \ln \frac{n_{ij}}{n_i/2} = 2N \ln 2 + 2 \sum_{i=1}^K \sum_{j=1}^2 n_{ij} \ln n_{ij} - 2 \sum_{i=1}^K n_i \ln n_i$$

where N is the total sum of all frequencies, i is the identification number (ID in Table 4), $j = 1, 2$ are the indices for left and right kanji respectively, $n_i = n_{i1} + n_{i2}$ (i.e. the marginal sum for the i th kanji) and $0 \ln 0 = 0$ by definition.

Using criterion (14) we obtain $X^2 = 16,720.30$ with 1,859 degree of freedom for the 1,945 kanji and using criterion (15) we obtain $2I = 2(76,485) \ln 2 + 2(138,221.6192 + 138,093.1164) - 2(319,903.0166) = 18,854.1622$. Then subtracting 1 for each zero in the table (complete Table 4 where there are 374 zeros) we obtain $2I = 18,483.16$ with 1,944 degree of freedom. Though the difference between the test results seems to be great, both of them indicated the same result³. The kanji productivity of compounds according to position before or behind the kanji is extremely significant, suggesting an 'asymmetric' pattern of kanji lexical productivity on both sides. Consequently, although individual kanji may display symmetry, there is no such symmetric tendency in the whole 1,945 basic kanji.

6. Summary

The present paper investigated the symmetry of the Japanese 1,945 basic kanji when producing two-kanji compound words. The results indicated the following two findings. First, construction of kanji compounds (or kanji lexical productivity) was represented by a birth-and-death process leading to the negative binomial and/or the Waring distribution. Second, although about a half of individual kanji displayed symmetry in lexical productivity on the left-hand and right-hand sides, there is no tendency on the whole for basic kanji to produce their compound words to the same extent on the left or the right side. In summary, lexical productivity of two-kanji compound words was highly asymmetric among the 1,945 basic kanji.

³ However, it is to be noted that this is not identical with the test for homogeneity for the left- and right-hand production.

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⁴ In this paper, including references, an alphabetic description of Japanese names follows the commonly used Hepburn style. As the Hepburn style does not distinguish between long and short vowels (for example, the proper name of ‘Kondo’ is pronounced /koNdoR/ with a long vowel at the end), this paper uses the spelling of ‘Kondo’, not ‘Kondoo’. However, to represent precise sounds, Japanese titles of research papers which include long vowels are shown by repeating the same vowels twice, such as ‘oo’.

Yokosawa, K., & Umeda, M. (1988). Processes in human Kanji-word recognition. *Proceedings of the 1988 IEEE international conference on systems, man, and cybernetics* (pp. 377-380). August 8-12, 1988, Beijing and Shenyang, China.

Appendixes

Table A

Observed and computed values of the formula (6) for kanji lexical productivity (frequency counts of two-kanji compounds) on the left-hand with class pooling

X[i]	F[i]	NP[i]	44	10	7.0600	89	2	2.0558
0	108	99.8942	45	9	6.8092	90	0	2.0049
1	135	109.1996	46	8	6.5738	91	1	1.9553
2	109	108.4107	47	5	6.3523	92	1	1.9069
3	128	104.0518	48	8	6.1436	93	2	1.8596
4	82	98.1938	49	6	5.9463	94	1	1.8135
5	84	91.7526	50	5	5.7596	95	0	1.7685
6	84	85.1954	51	2	5.5824	96	2	1.7247
7	61	78.7763	52	7	5.4141	97	1	1.6818
8	73	72.6353	53	9	5.2537	98	1	1.6401
9	67	66.8469	54	3	5.1008	99	1	1.5993
10	59	61.4462	55	6	4.9546	100	1	1.5596
11	53	56.4438	56	2	4.8147	101	1	1.5208
12	48	51.8353	57	5	4.6805	102	1	1.4829
13	49	47.6072	58	3	4.5517	103	1	1.4460
14	36	43.7403	59	5	4.4279	104	1	1.4099
15	37	40.2123	60	3	4.3087	105	0	1.3748
16	37	36.9996	61	3	4.1939	106	4	1.3404
17	39	34.0784	62	3	4.0830	107	0	1.3070
18	33	31.4251	63	3	3.9760	108	1	1.2743
19	29	29.0171	64	6	3.8725	109	2	1.2425
20	17	26.8330	65	6	3.7723	110	1	1.2114
21	34	24.8526	66	3	3.6753	111	1	1.1811
22	28	23.0571	67	8	3.5813	112	1	1.1515
23	15	21.4292	68	3	3.4902	113	0	1.1226
24	17	19.9530	69	3	3.4017	114	1	1.0945
25	22	18.6137	70	3	3.3158	115	0	1.0670
26	21	17.3981	71	2	3.2323	116	1	1.0402
27	19	16.2939	72	1	3.1512	117	1	1.0141
28	16	15.2902	73	1	3.0723	118	4	0.9886
29	13	14.3770	74	0	2.9955	119	0	0.9637
30	16	13.5451	75	5	2.9209	120	2	0.9395
31	15	12.7865	76	1	2.8482	121	2	0.9158
32	13	12.0937	77	3	2.7774	122	1	0.8928
33	8	11.4602	78	2	2.7084	123	1	0.8703
34	11	10.8800	79	2	2.6412	124	0	0.8483
35	10	10.3476	80	2	2.5758	125	0	0.8269
36	14	9.8584	81	0	2.5120	126	1	0.8060
37	14	9.4079	82	4	2.4498	127	1	0.7857
38	16	8.9923	83	2	2.3892	128	1	0.7658
39	5	8.6081	84	2	2.3301	129	1	0.7464
40	9	8.2522	85	2	2.2725	130	1	0.7276
41	13	7.9218	86	1	2.2163	131	2	0.7091
42	6	7.6144	87	1	2.1614	132	1	0.6912
43	8	7.3278	88	1	2.1080	133	0	0.6737

134	0	0.6566	179	0	0.2046	224	0	0.0628
135	2	0.6400	180	1	0.1994	225	0	0.0612
136	1	0.6237	181	0	0.1942	226	0	0.0596
137	0	0.6079	182	0	0.1892	227	0	0.0581
138	1	0.5924	183	0	0.1843	228	0	0.0566
139	0	0.5774	184	0	0.1796	229	0	0.0551
140	1	0.5627	185	0	0.1749	230	0	0.0536
141	0	0.5484	186	0	0.1704	231	0	0.0522
142	1	0.5345	187	0	0.1660	232	0	0.0509
143	0	0.5209	188	0	0.1617	233	0	0.0496
144	0	0.5076	189	0	0.1576	234	0	0.0483
145	0	0.4947	190	0	0.1535	235	0	0.0470
146	1	0.4821	191	0	0.1495	236	0	0.0458
147	0	0.4698	192	0	0.1457	237	0	0.0446
148	1	0.4578	193	0	0.1419	238	0	0.0434
149	0	0.4461	194	0	0.1382	239	0	0.0423
150	0	0.4347	195	0	0.1347	240	0	0.0412
151	1	0.4236	196	0	0.1312	241	0	0.0401
152	0	0.4128	197	0	0.1278	242	0	0.0391
153	1	0.4023	198	0	0.1245	243	0	0.0381
154	0	0.3920	199	0	0.1213	244	0	0.0371
155	0	0.3820	200	0	0.1181	245	0	0.0361
156	0	0.3722	201	0	0.1151	246	1	0.0352
157	0	0.3627	202	0	0.1121	247	0	0.0342
158	0	0.3534	203	0	0.1092	248	0	0.0333
159	0	0.3443	204	0	0.1063	249	0	0.0325
160	0	0.3355	205	0	0.1036	250	0	0.0316
161	0	0.3269	206	0	0.1009	251	0	0.0308
162	1	0.3185	207	0	0.0983	252	0	0.0300
163	0	0.3104	208	0	0.0957	253	0	0.0292
164	0	0.3024	209	0	0.0933	254	0	0.0284
165	0	0.2947	210	0	0.0908	255	0	0.0277
166	0	0.2871	211	0	0.0885	256	0	0.0270
167	0	0.2797	212	0	0.0862	257	0	0.0263
168	0	0.2725	213	1	0.0839	258	0	0.0256
169	0	0.2655	214	0	0.0818	259	0	0.0249
170	1	0.2587	215	0	0.0796	260	0	0.0243
171	1	0.2521	216	0	0.0776	261	0	0.0236
172	0	0.2456	217	0	0.0756	262	0	0.0230
173	0	0.2393	218	0	0.0736	263	0	0.0224
174	0	0.2331	219	0	0.0717	264	0	0.0218
175	0	0.2271	220	0	0.0698	265	0	0.0213
176	0	0.2213	221	0	0.0680	266	0	0.0207
177	0	0.2156	222	0	0.0662	267	0	0.0202
178	0	0.2100	223	0	0.0645	268	1	0.7476

Table B

Fitting the formula (6) to kanji lexical productivity on the right-hand side

X[i]	F[i]	NP[i]	52	6	6.1783	105	0	1.3891
0	72	70.0911	53	4	6.0156	106	3	1.3481
1	91	94.3658	54	8	5.8576	107	1	1.3082
2	115	103.6675	55	4	5.7040	108	0	1.2694
3	121	105.3928	56	2	5.5545	109	1	1.2317
4	90	102.8910	57	4	5.4089	110	1	1.1950
5	93	98.0122	58	1	5.2670	111	2	1.1594
6	92	91.8696	59	6	5.1288	112	0	1.1248
7	87	85.1598	60	3	4.9939	113	0	1.0912
8	80	78.3235	61	5	4.8624	114	1	1.0585
9	61	71.6374	62	4	4.7340	115	0	1.0268
10	78	65.2700	63	5	4.6087	116	1	0.9959
11	51	59.3174	64	3	4.4864	117	0	0.9660
12	57	53.8274	65	0	4.3670	118	1	0.9369
13	49	48.8154	66	1	4.2504	119	1	0.9087
14	51	44.2755	67	3	4.1366	120	1	0.8813
15	34	40.1885	68	1	4.0255	121	0	0.8546
16	45	36.5269	69	2	3.9170	122	0	0.8288
17	32	33.2588	70	2	3.8112	123	0	0.8037
18	35	30.3507	71	4	3.7078	124	0	0.7793
19	31	27.7686	72	5	3.6070	125	0	0.7556
20	29	25.4797	73	3	3.5085	126	1	0.7326
21	22	23.4529	74	1	3.4125	127	1	0.7103
22	21	21.6591	75	3	3.3188	128	0	0.6887
23	18	20.0716	76	2	3.2273	129	0	0.6677
24	22	18.6662	77	6	3.1381	130	0	0.6473
25	11	17.4210	78	1	3.0512	131	0	0.6275
26	21	16.3162	79	3	2.9663	132	1	0.6083
27	15	15.3343	80	4	2.8836	133	0	0.5896
28	17	14.4599	81	0	2.8030	134	0	0.5715
29	12	13.6792	82	1	2.7244	135	2	0.5540
30	14	12.9801	83	2	2.6478	136	0	0.5370
31	15	12.3521	84	0	2.5732	137	2	0.5204
32	15	11.7858	85	0	2.5004	138	3	0.5044
33	17	11.2733	86	1	2.4295	139	0	0.4889
34	16	10.8074	87	3	2.3605	140	0	0.4738
35	7	10.3820	88	1	2.2933	141	0	0.4591
36	13	9.9920	89	0	2.2278	142	0	0.4450
37	13	9.6326	90	2	2.1640	143	0	0.4312
38	13	9.2999	91	1	2.1019	144	0	0.4178
39	7	8.9906	92	1	2.0415	145	1	0.4049
40	11	8.7016	93	0	1.9826	146	1	0.3923
41	11	8.4305	94	0	1.9254	147	0	0.3801
42	15	8.1751	95	2	1.8696	148	1	0.3683
43	9	7.9334	96	1	1.8154	149	1	0.3569
44	12	7.7040	97	1	1.7626	150	0	0.3458
45	4	7.4854	98	0	1.7113	151	1	0.3350
46	6	7.2764	99	1	1.6613	152	0	0.3245
47	10	7.0762	100	0	1.6127	153	0	0.3144
48	5	6.8837	101	1	1.5655	154	2	0.3046
49	7	6.6982	102	0	1.5195	155	0	0.2951
50	8	6.5192	103	2	1.4748	156	1	0.2859
51	5	6.3460	104	1	1.4314	157	1	0.2769

158	1	0.2682	215	0	0.0424	272	0	0.0064
159	0	0.2598	216	0	0.0410	273	0	0.0062
160	1	0.2517	217	0	0.0397	274	0	0.0060
161	0	0.2438	218	0	0.0384	275	0	0.0058
162	0	0.2361	219	0	0.0372	276	0	0.0056
163	1	0.2287	220	0	0.0360	277	0	0.0054
164	0	0.2215	221	0	0.0348	278	0	0.0053
165	0	0.2145	222	0	0.0337	279	0	0.0051
166	0	0.2078	223	0	0.0326	280	0	0.0049
167	0	0.2012	224	0	0.0315	281	0	0.0048
168	0	0.1949	225	0	0.0305	282	0	0.0046
169	0	0.1887	226	0	0.0295	283	0	0.0044
170	1	0.1828	227	0	0.0286	284	0	0.0043
171	0	0.1770	228	0	0.0276	285	0	0.0042
172	0	0.1714	229	0	0.0267	286	0	0.0040
173	0	0.1660	230	0	0.0259	287	0	0.0039
174	0	0.1607	231	0	0.0250	288	0	0.0038
175	0	0.1557	232	0	0.0242	289	0	0.0036
176	0	0.1507	233	0	0.0234	290	0	0.0035
177	0	0.1459	234	0	0.0227	291	0	0.0034
178	0	0.1413	235	0	0.0219	292	0	0.0033
179	1	0.1368	236	0	0.0212	293	1	0.0032
180	0	0.1325	237	0	0.0205	294	0	0.0031
181	0	0.1283	238	0	0.0199	295	0	0.0030
182	0	0.1242	239	0	0.0192	296	0	0.0029
183	0	0.1202	240	0	0.0186	297	0	0.0028
184	0	0.1164	241	0	0.0180	298	0	0.0027
185	0	0.1127	242	0	0.0174	299	0	0.0026
186	0	0.1091	243	0	0.0168	300	0	0.0025
187	0	0.1056	244	0	0.0163	301	0	0.0024
188	0	0.1023	245	0	0.0158	302	0	0.0024
189	0	0.0990	246	0	0.0152	303	0	0.0023
190	0	0.0959	247	0	0.0148	304	0	0.0022
191	0	0.0928	248	0	0.0143	305	0	0.0021
192	0	0.0898	249	0	0.0138	306	0	0.0021
193	0	0.0870	250	0	0.0134	307	0	0.0020
194	1	0.0842	251	0	0.0129	308	0	0.0019
195	1	0.0815	252	0	0.0125	309	0	0.0019
196	0	0.0789	253	0	0.0121	310	0	0.0018
197	0	0.0763	254	0	0.0117	311	0	0.0017
198	0	0.0739	255	0	0.0113	312	0	0.0017
199	0	0.0715	256	0	0.0109	313	0	0.0016
200	0	0.0692	257	0	0.0106	314	0	0.0016
201	0	0.0670	258	0	0.0102	315	0	0.0015
202	0	0.0649	259	0	0.0099	316	0	0.0015
203	0	0.0628	260	0	0.0096	317	0	0.0014
204	0	0.0608	261	0	0.0093	318	0	0.0014
205	0	0.0588	262	0	0.0090	319	0	0.0013
206	0	0.0569	263	0	0.0087	320	0	0.0013
207	0	0.0551	264	0	0.0084	321	0	0.0012
208	0	0.0533	265	0	0.0081	322	0	0.0012
209	0	0.0516	266	0	0.0078	323	0	0.0012
210	0	0.0499	267	0	0.0076	324	0	0.0011
211	0	0.0483	268	0	0.0073	325	0	0.0011
212	0	0.0468	269	0	0.0071	326	0	0.0010
213	0	0.0453	270	0	0.0069	327	0	0.0010
214	0	0.0438	271	0	0.0066	328	0	0.0010

329	0	0.0009	353	0	0.0004	377	0	0.0002
330	0	0.0009	354	0	0.0004	378	0	0.0002
331	0	0.0009	355	0	0.0004	379	0	0.0002
332	0	0.0009	356	0	0.0004	380	0	0.0002
333	0	0.0008	357	0	0.0004	381	0	0.0002
334	0	0.0008	358	0	0.0004	382	0	0.0002
335	0	0.0008	359	0	0.0003	383	0	0.0002
336	0	0.0007	360	0	0.0003	384	0	0.0001
337	0	0.0007	361	0	0.0003	385	0	0.0001
338	0	0.0007	362	0	0.0003	386	0	0.0001
339	0	0.0007	363	0	0.0003	387	0	0.0001
340	0	0.0007	364	0	0.0003	388	0	0.0001
341	0	0.0006	365	0	0.0003	389	0	0.0001
342	0	0.0006	366	0	0.0003	390	0	0.0001
343	0	0.0006	367	0	0.0003	391	0	0.0001
344	0	0.0006	368	0	0.0003	392	0	0.0001
345	0	0.0006	369	0	0.0002	393	0	0.0001
346	0	0.0005	370	0	0.0002	394	0	0.0001
347	0	0.0005	371	0	0.0002	395	0	0.0001
348	0	0.0005	372	0	0.0002	396	0	0.0001
349	0	0.0005	373	0	0.0002	397	0	0.0001
350	1	0.0005	374	0	0.0002	398	0	0.0001
351	0	0.0005	375	0	0.0002	399	1	0.0026
352	0	0.0004	376	0	0.0002			

Table C

The intervals for the symmetry of compound building

n	k1	k2	34, [11, 23]	63, [24, 39]	92, [37, 55]
6, [1, 5]			35, [12, 23]	64, [24, 40]	93, [37, 56]
7, [1, 6]			36, [12, 24]	65, [25, 40]	94, [38, 56]
8, [1, 7]			37, [13, 24]	66, [25, 41]	95, [38, 57]
9, [2, 7]			38, [13, 25]	67, [26, 41]	96, [38, 58]
10, [2, 8]			39, [13, 26]	68, [26, 42]	97, [39, 58]
11, [2, 9]			40, [14, 26]	69, [26, 43]	98, [39, 59]
12, [3, 9]			41, [14, 27]	70, [27, 43]	99, [40, 59]
13, [3, 10]			42, [15, 27]	71, [27, 44]	100, [40, 60]
14, [3, 11]			43, [15, 28]	72, [28, 44]	101, [41, 60]
15, [4, 11]			44, [16, 28]	73, [28, 45]	102, [41, 61]
16, [4, 12]			45, [16, 29]	74, [29, 45]	103, [42, 61]
17, [5, 12]			46, [16, 30]	75, [29, 46]	104, [42, 62]
18, [5, 13]			47, [17, 30]	76, [29, 47]	105, [42, 63]
19, [5, 14]			48, [17, 31]	77, [30, 47]	106, [43, 63]
20, [6, 14]			49, [18, 31]	78, [30, 48]	107, [43, 64]
21, [6, 15]			50, [18, 32]	79, [31, 48]	108, [44, 64]
22, [6, 16]			51, [19, 32]	80, [31, 49]	109, [44, 65]
23, [7, 16]			52, [19, 33]	81, [32, 49]	110, [45, 65]
24, [7, 17]			53, [19, 34]	82, [32, 50]	111, [45, 66]
25, [8, 17]			54, [20, 34]	83, [33, 50]	112, [46, 66]
26, [8, 18]			55, [20, 35]	84, [33, 51]	113, [46, 67]
27, [8, 19]			56, [21, 35]	85, [33, 52]	114, [47, 67]
28, [9, 19]			57, [21, 36]	86, [34, 52]	115, [47, 68]
29, [9, 20]			58, [22, 36]	87, [34, 53]	116, [47, 69]
30, [10, 20]			59, [22, 37]	88, [35, 53]	117, [48, 69]
31, [10, 21]			60, [22, 38]	89, [35, 54]	118, [48, 70]
32, [10, 22]			61, [23, 38]	90, [36, 54]	119, [49, 70]
33, [11, 22]			62, [23, 39]	91, [36, 55]	120, [49, 71]

121, [50, 71]	178, [76, 102]	235, [102, 133]	292, [129, 163]
122, [50, 72]	179, [76, 103]	236, [103, 133]	293, [130, 163]
123, [51, 72]	180, [77, 103]	237, [103, 134]	294, [130, 164]
124, [51, 73]	181, [77, 104]	238, [104, 134]	295, [131, 164]
125, [52, 73]	182, [78, 104]	239, [104, 135]	296, [131, 165]
126, [52, 74]	183, [78, 105]	240, [105, 135]	297, [132, 165]
127, [52, 75]	184, [79, 105]	241, [105, 136]	298, [132, 166]
128, [53, 75]	185, [79, 106]	242, [106, 136]	299, [133, 166]
129, [53, 76]	186, [80, 106]	243, [106, 137]	300, [133, 167]
130, [54, 76]	187, [80, 107]	244, [107, 137]	301, [134, 167]
131, [54, 77]	188, [81, 107]	245, [107, 138]	302, [134, 168]
132, [55, 77]	189, [81, 108]	246, [108, 138]	303, [134, 169]
133, [55, 78]	190, [82, 108]	247, [108, 139]	304, [135, 169]
134, [56, 78]	191, [82, 109]	248, [109, 139]	305, [135, 170]
135, [56, 79]	192, [82, 110]	249, [109, 140]	306, [136, 170]
136, [57, 79]	193, [83, 110]	250, [110, 140]	307, [136, 171]
137, [57, 80]	194, [83, 111]	251, [110, 141]	308, [137, 171]
138, [58, 80]	195, [84, 111]	252, [110, 142]	309, [137, 172]
139, [58, 81]	196, [84, 112]	253, [111, 142]	310, [138, 172]
140, [58, 82]	197, [85, 112]	254, [111, 143]	311, [138, 173]
141, [59, 82]	198, [85, 113]	255, [112, 143]	312, [139, 173]
142, [59, 83]	199, [86, 113]	256, [112, 144]	313, [139, 174]
143, [60, 83]	200, [86, 114]	257, [113, 144]	314, [140, 174]
144, [60, 84]	201, [87, 114]	258, [113, 145]	315, [140, 175]
145, [61, 84]	202, [87, 115]	259, [114, 145]	316, [141, 175]
146, [61, 85]	203, [88, 115]	260, [114, 146]	317, [141, 176]
147, [62, 85]	204, [88, 116]	261, [115, 146]	318, [142, 176]
148, [62, 86]	205, [88, 117]	262, [115, 147]	319, [142, 177]
149, [63, 86]	206, [89, 117]	263, [116, 147]	320, [142, 178]
150, [63, 87]	207, [89, 118]	264, [116, 148]	321, [143, 178]
151, [63, 88]	208, [90, 118]	265, [117, 148]	322, [143, 179]
152, [64, 88]	209, [90, 119]	266, [117, 149]	323, [144, 179]
153, [64, 89]	210, [91, 119]	267, [118, 149]	324, [144, 180]
154, [65, 89]	211, [91, 120]	268, [118, 150]	325, [145, 180]
155, [65, 90]	212, [92, 120]	269, [118, 151]	326, [145, 181]
156, [66, 90]	213, [92, 121]	270, [119, 151]	327, [146, 181]
157, [66, 91]	214, [93, 121]	271, [119, 152]	328, [146, 182]
158, [67, 91]	215, [93, 122]	272, [120, 152]	329, [147, 182]
159, [67, 92]	216, [94, 122]	273, [120, 153]	330, [147, 183]
160, [68, 92]	217, [94, 123]	274, [121, 153]	331, [148, 183]
161, [68, 93]	218, [95, 123]	275, [121, 154]	332, [148, 184]
162, [69, 93]	219, [95, 124]	276, [122, 154]	333, [149, 184]
163, [69, 94]	220, [95, 125]	277, [122, 155]	334, [149, 185]
164, [69, 95]	221, [96, 125]	278, [123, 155]	335, [150, 185]
165, [70, 95]	222, [96, 126]	279, [123, 156]	336, [150, 186]
166, [70, 96]	223, [97, 126]	280, [124, 156]	337, [151, 186]
167, [71, 96]	224, [97, 127]	281, [124, 157]	338, [151, 187]
168, [71, 97]	225, [98, 127]	282, [125, 157]	339, [151, 188]
169, [72, 97]	226, [98, 128]	283, [125, 158]	340, [152, 188]
170, [72, 98]	227, [99, 128]	284, [125, 159]	341, [152, 189]
171, [73, 98]	228, [99, 129]	285, [126, 159]	342, [153, 189]
172, [73, 99]	229, [100, 129]	286, [126, 160]	343, [153, 190]
173, [74, 99]	230, [100, 130]	287, [127, 160]	344, [154, 190]
174, [74, 100]	231, [101, 130]	288, [127, 161]	345, [154, 191]
175, [75, 100]	232, [101, 131]	289, [128, 161]	346, [155, 191]
176, [75, 101]	233, [102, 131]	290, [128, 162]	347, [155, 192]
177, [75, 102]	234, [102, 132]	291, [129, 162]	348, [156, 192]

349, [156, 193]	362, [162, 200]	375, [169, 206]	388, [175, 213]
350, [157, 193]	363, [163, 200]	376, [169, 207]	389, [175, 214]
351, [157, 194]	364, [163, 201]	377, [169, 208]	390, [176, 214]
352, [158, 194]	365, [164, 201]	378, [170, 208]	391, [176, 215]
353, [158, 195]	366, [164, 202]	379, [170, 209]	392, [177, 215]
354, [159, 195]	367, [165, 202]	380, [171, 209]	393, [177, 216]
355, [159, 196]	368, [165, 203]	381, [171, 210]	394, [178, 216]
356, [160, 196]	369, [166, 203]	382, [172, 210]	395, [178, 217]
357, [160, 197]	370, [166, 204]	383, [172, 211]	396, [179, 217]
358, [160, 198]	371, [167, 204]	384, [173, 211]	397, [179, 218]
359, [161, 198]	372, [167, 205]	385, [173, 212]	398, [179, 219]
360, [161, 199]	373, [168, 205]	386, [174, 212]	399, [180, 219]
361, [162, 199]	374, [168, 206]	387, [174, 213]	400, [180, 220]